

Political Science 152/352
Fall 2002
Problem Set 1
Due Wednesday, Oct. 9, in class.

1. Three friends adopt the following procedure for choosing one of three restaurants to go to for dinner. Simultaneously, each writes the name of one of the three restaurants on a piece of paper, and then they put the pieces of paper in a hat. One piece is then drawn from the hat at random.
 - (a) Represent this interaction as an extensive form game.
 - (b) Neglecting the moves by Nature, what are the dimensions of the normal form version of this game? Use set notation to represent S_i , and provide an example of an element of the set $S = S_1 \times S_2 \times S_3$.
 - (c) Now suppose that two of the friends choose simultaneously, and the third observes their choices before making hers. Draw the extensive form representation of the game. What are the dimensions of its normal form representation (omitting the moves by Nature)?
 - (d) (bonus) For the game in (a), can you think of any circumstance in which a player would *not* write down his or her first preference for a restaurant? (Assume each is trying to maximize the chance of getting his or her first preference.)
2. Suppose that players 1 and 2 play the State of Nature game twice in a row, and that they observe what each chose in the first “round” before they make their choices for the second round.
 - (a) Draw the extensive form version of this game.
 - (b) What are the dimensions of the corresponding normal form? Give an example of a complete strategy for player 1 and a complete strategy for player 2 in this game, and describe the outcome of the game when the players choose these strategies.
3. In a certain game model of the political economy of dictatorship, the dictator in the game chooses at once a tax rate and whether or not to enforce compliance. Use set and cartesian product notation to characterize the set of strategies available to the dictator, and give two examples of specific strategies that are elements of this set.
4. In a game model of states’ decisions to arm, two states simultaneously choose what share of their national income to allocate to arms (“defense”). Use set notation to represent the players’ strategy sets and the set of strategy combinations (i.e. outcomes) for this game, and give two examples of elements of the set of outcomes. Graph what the set of outcomes looks like in \mathbb{R}^2 .
5. In a common experimental game, n players simultaneously choose a number between zero and 100, and the winner is the player whose choice is closest to 70% of the average of all the choices.
 - (a) Use set notation to characterize an individual’s strategy set and the set of all possible outcomes. Give two examples of a possible outcome for the case of $n = 5$.
 - (b) A strategy is called *weakly dominated* for a player if there is some other strategy that always gives the player at least as good an outcome and in some cases gives a better outcome, no matter what the other players choose. Identify a weakly dominated strategy in this game and explain why it is weakly dominated and what other strategy weakly dominates it.

6. Two factions form in the State of Nature, each with a leader who would like to be monarch over all. Characterize a game between the two leaders in which each chooses simultaneously whether to demand rule for himself or to cede rule to the other. Assume that if they agree, a government forms under the leader who demanded rule. If they disagree, either because both demanded rule or neither did, then assume that the outcome is a return to the State of Nature for both factions. Finally, assume that while government of any sort is better than the State of Nature, ruling is better than being ruled. Characterize and analyze the game implied by this description along the lines of the examples done in the first two classes (i.e., specify plausible preferences and best replies, and discuss in what ways the problem is similar to and different from the social “contract” game discussed in class).