Political Science 152/352
Fall 2002
Problem Set 2
Due Friday, Oct. 18.

1. Show that asymmetry and negative transitivity of the strong preference relation imply transitivity of the strong preference relation. Extra credit: Show that preferences that satisfy transitivity of the strong preference relation need not satisfy negative transitivity.

2. Consider the following three sets of policy outcomes:

$$G = \{g_1 = \text{status quo on gun control}, g_2 = \text{much stronger gun control}\}$$

$$F = \{f_1 = \text{U.S. invades Iraq without a supportive U.N. resolution}, f_2 = \text{U.S. does not invade Iraq}\}$$

$$A = \{a_1 = \text{status quo on abortion policy}, a_2 = \text{overturning of Roe vs. Wade}\}$$

   (a) Define your preferences ($\succ$) on G, F and A.
   (b) Define your preferences ($\succ$) on $X = G \times F \times A$.
   (c) Give two ordinal utility functions that represent your preferences on X.
   (d) Give a utility function $u : X \to R$, representing the preferences of a person who cares *only* about abortion policy, and does not care at all about the other issues.
   (e) A policy outcome is "Pareto-efficient" for a set of decision-makers if there is no alternative that would make everyone in the set at least as well off and make at least one person in the set strictly better off. Considering your preferences in (b) and the preferences of the person in (d), what is the set of Pareto-efficient policies from X?

3. You are new in town, your phone doesn't work yet, the gas for your stove hasn't been turned on yet, there are two restaurants in the town, it is Monday night, and you are hungry. You have been told that restaurant A is better than restaurant B. Suppose that you choose a restaurant to try first, and if it is closed you will go to the other one. Suppose you estimate that the probability that A is open is p and the probability that B is open is q. Further, suppose your preferences are representable by the following expected utility function: Your utility for eating at A is $a > 0$; your utility for eating at B is αa, where $\alpha \in (0, 1)$. Your utility for not getting dinner is 0, and you suffer a cost of delay and effort for going from one restaurant to the other if the first is closed of c. (So, for example, if you go to A first and find it is closed, then go to B, which is open, you "get" $\alpha a - c$ in utility terms.) Assume further that c is small in relation to the utility for eating a meal at either restaurant.

   (a) Under what conditions on p and q will you prefer going to A first rather than B, if you can be represented as an expected utility maximizer?
   (b) What does it mean in terms of your preferences to decrease α, and how do interpret the effect of doing so on your preferences concerning which restaurant to try first?
   (c) Show that changing the utility function by multiplying by $K > 0$ and adding M has no effect on the condition you derived in (i).

4. The time has come to construct your own von Neumann-Morgenstern utility function over grades. Let $X = \{A, A-, B+, B, B-\}$.

   (a) Define an ordinal utility function that represents your preferences over these outcomes, $u : X \to \mathbb{R}$.

(b) Consider the set of gambles on X, $\Delta(X)$. Decide the following *by introspection.* What probability t would make you indifferent between

i. $p = (t, 0, 0, 0, 1 - t)$ and $q = (0, 0, 0, 1, 0)$?

ii. $p = (t, 0, 0, 0, 1 - t)$ and $q = (0, 0, 1, 0, 0)$?

iii. $p = (t, 0, 0, 0, 1 - t)$ and $q = (0, 1, 0, 0, 0)$?

State the probability that you think you would actually require if you faced this problem – *no mathematical calculations are needed here.*

(c) Use your answers in (b) to construct an expected utility function representing your preferences over X.

(d) Answer the following *by introspection*: What probability t would make you indifferent between $p = (0, t, 0, 1 - t, 0)$ and $q = (0, 0, 1, 0, 0)$?

(e) Using your answer to (e) and your utility function from (d), calculate your expected utility for the gamble $p = (0, 1 - t, 0, t, 0)$. Is the result consistent or inconsistent with your utility function?

5. It is common to talk about politicians being "to the left" or "to the right" or "at the center," and it is common for political scientists to represent this idea of a left-right political "dimension" by using the number line $\mathbb{R}$. Suppose we identify different left-right positions on policies with numbers on this line. Consider a voter with an "ideal point" (most preferred policy position) at 0. Suppose the voter is trying to choose which of two political candidates to vote for, A or B, and is unsure about the true issue position of each candidate. The voter believes that A's true position is equally likely to be either $-a$, 0, or a, where $a > 0$. Likewise, the voter thinks that B's true position is equally likely to be either $-b$, 0, or b, where $b > a$.

(a) What is the expected issue position of candidates A and B? (Show how this works out mathematically.)

(b) Suppose that the voters preferences over positions in the issue space can be represented by the expected utility function $u(x) = -|x|$. Will the voter prefer to vote for A or B?

(c) Repeat (ii) assuming that the voter's preferences are represented by $v(x) = -x^2$. Interpret what is going on.

(d) Politicians sometimes seem intentionally to make their positions on issues unclear, or to refuse to clarify their positions. The standard argument holds that they do this so as not to alienate voters – they are trying to be "all things to all people." What is puzzling about the above in light of this empirical observation and conjecture?

6. Assign generic vNM utility numbers for two players in the Social Contract game. Find the probability v^* that 2 plays V such that 1 is indifferent between choosing V and NV. How does v^* vary as you lower the payoff for the "war at disadvantage for 1" outcome, and why? What is 1's best response (i.e. choice of V or NV) for all values of $v \in [0, 1]$?