

Political Science 152/352
 Fall 2002
 Problem Set 3
 Due Wednesday, November 6, 2002, in class.

1. Consider the following normal form game.

		Person 2		
		<i>L</i>	<i>C</i>	<i>R</i>
Person 1	<i>U</i>	10, 0	0, 10	-1, 6
	<i>M</i>	0, 10	10, 0	-1, 6
	<i>D</i>	6, -1	6, -1	2, 2

- (a) What is the set of rationalizable pure-strategy profiles for this game?
 - (b) What is the set of Nash equilibria of this game (pure and mixed)?
2. Two candidates for office simultaneously choose policy positions in an issue space defined by $X = \{0, 1, 2, \dots, 100\}$. There are 101 voters in their district, with ideal points at each of the 101 positions in X . Voters automatically vote for the candidate who is located closest to his or her ideal point. If a voter is indifferent between the two candidates, he gives 1/2 vote to each (you could also think of this as the expected outcome of a 50-50 randomization). A candidate gets a payoff of 1 if she wins the election, 0 if she loses, and 1/2 if there is a tie.
 - (a) Formulate this problem as a two player normal form game (i.e., treat the voters as “parametric,” i.e., not as players but as automatically incorporated in the utility functions of the candidates).
 - (b) Does this game have any strictly dominated strategies? If so, how do you know?
 - (c) Solve this game using iterated deletion of strictly dominated strategies. Note that you will have to explain your reasoning rather than drawing the 101×101 normal form!
 - (d) This game has a unique Nash equilibrium. Find it, and show that it is the unique equilibrium in pure strategies.
 - (e) From a normative perspective, do you think this outcome is a good thing or a bad thing?
 3. Consider the following simple game. Army A has a single plane with which it can strike one of two possible targets. Army B has one anti-aircraft gun that can be assigned to one of the targets. The value of target 1 is v_1 and the value of target 2 is 1, where $v_1 > 1$. Army A can destroy a target only if the target is undefended and A attacks it (otherwise, if A attacks a defended target nothing happens and payoffs are (0, 0)). Army A wishes to maximize the expected value of the damage and B wishes to minimize it.
 - (a) Formulate this problem as a normal-form game and find its unique Nash equilibrium.

- (b) How does increasing the value of target 1 affect the probability that it will be destroyed? How does this affect the risk that target 2 will be destroyed? Explain the logic of what is happening in the equilibrium to produce these results.
4. 100,000 people (apart from members of the government) live in the capitol city of Megalomania, an impoverished dictatorship. These people choose simultaneously whether to demand the fall of the government by protesting in the main square of the city, or to stay at home. If at least 10% of them protest, then the government collapses due to internal defections (this is known by all). If, however, fewer than 10% protest, then the government remains, and some of the protesters are arrested and jailed. The police can jail 600 people, so that the probability of being jailed is 1 if 600 or fewer turn out, and $600/x$ if x people turn out where x is greater than 600 but less than 10%. If the government falls, no protester is jailed.

Every individual in the city values the status quo at 0 and a collapse of the regime at 10. The utility for being arrested, however, is $-k$ where $k > 10$.

- (a) Assume that for all individuals the act of protesting is itself a bit costly in terms of time and effort. Assume in particular that every protester suffers a utility cost of 1 for protesting, regardless of whether he or she is arrested and regardless of whether the regime falls or not. Set up the problem as a normal form game, find any or all pure-strategy Nash equilibria, and interpret your results (i.e., explain “what is going on” and why). What kind or kinds of strategic problem is this?
- (b) Now assume that for all individuals the act of protesting is actually a positive thing, either because it is exciting and fun or because the act of expressing dissent against the hated regime is itself enjoyable or both. Assume that all protesters get a utility benefit of 1 for protesting. Set up the problem as a normal form game, find any or all pure-strategy Nash equilibria, and interpret your results (i.e., explain “what is going on” and why). Contrast your findings to (a).
- (c) It is commonly observed that large-scale protests in the capitol cities of authoritarian regimes tend to occur on the anniversaries of other protests or striking acts of resistance or oppression. Explain why in light of the model here and concepts introduced in the course.
5. *A contest model.* Two candidates simultaneously choose how much money to raise and spend in a congressional campaign. The value of winning the seat (in utility) is v for both, and the probability that candidate 1 wins is $p(s_1, s_2) = s_1/(s_1 + s_2)$, where $s_i \geq 0$ is how much candidate $i = 1, 2$ raises and spends on the race. (Assume $p(0, 0) = 1/2$ to be complete.) The utility cost of raising funds is $k_1 s_1$ for candidate 1, and $k_2 s_2$ for candidate 2, where k_1 and k_2 are positive constants that represent how distasteful or difficult it is for the candidate to raise money, relative to the prize of winning. (For example, a natural hypothesis is that incumbents find it much easier to raise money, and so have lower k_i 's.)
- (a) Formulate this problem as a normal form game (basically, this means specify the utility functions here, since the rest is pretty much done).
- (b) If you know calculus, find 1's best reply function – that is, the function that says what 1's best response is as a function of spending by candidate 2. Do the same for candidate 2, and then solve for the Nash equilibrium.
- (c) If you can, set $k_2 = 1$ and then do and interpret comparative statics on k_1 . If “ k_1 small, $k_2 = 1$ represents a contest between an incumbent and a challenger, and $k_1 = k_2 = 1$ is an open seat race, what is predicted concerning spending levels and chances of winning?”