Political Science 152/352

Fall 2002

Problem Set

Due Monday, November 25, in class.

1. What is the appropriate game theoretic solution concept for tic-tac-toe? For chess? Use this observation to argue that chess "has a solution."

2. Two people bargain over the division of three units of an indivisible good, using the following protocol. Player 1 makes an offer to player 2, which is an integer between 0 and 3 (including 0 and 3). Player 2 observes the offer, and then says Yes or No. If Yes, then the players' payoffs are x for player 1 and $3 - x$ for player 2, where x was 1's offer/demand. If No, then both players receive a payoff of zero.

   (a) Draw the extensive form of this game, with payoffs, and find any or all subgame perfect Nash equilibria. Choose the SGPNE that you find most intuitive and explain what is happening in it in ordinary language.

   (b) Draw the normal form of the game, and then find all Nash equilibria.

   (c) Choose a Nash equilibrium that is not subgame perfect and explain what is happening in this equilibrium in ordinary language.

3. A government agency, A, will propose a budget level $b \geq 0$ to a legislature that we will represent by considering the "median legislator." The legislator has a most preferred budget level b_m, with preferences that can be represented with utility function $u_m(b) = -(b - b_m)^2$.

   After b has been proposed, the median legislator decides whether to approve the budget level b or not to. If she rejects b then a "reversion level" $b^* \geq 0$ is implemented. Assume that the agency always prefers a bigger budget, and in particular, that $u_A(b) = b$.

   (a) Formally specify a strategy for the agency and a strategy of the legislator. (If you can not do this formally, use an equivalent verbal description.)

   (b) Solve the game using subgame-perfect Nash equilibrium as your solution concept. To simplify the problem somewhat, consider first, the case in which $b_m \leq b^*$ and second in which $b_m > b^*$.

   (c) Plot the equilibrium budget level as a function of the reversion level b^*. Interpret what is going on. How and when does the agency's structural bargaining power matter?

4. Two superpowers face the problem of avoiding nuclear war as they jockey for influence around the globe. We represent their problem as an infinitely repeated game of "Chicken," with the stage-game payoffs described below ($\alpha > 1, \beta > 0$):

	Abide	Infringe
Abide	1, 1	0, α
Infringe	α, 0	$-\beta, -\beta$

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"Abide" means to abide by some understood rules concerning "spheres of influence." "Infringe" means to encroach on the other side's sphere of influence. If both encroach, there is a risk of nuclear war which is bad for both. Suppose that in the repeated game they both have a share a common discount factor of $\delta \in (0, 1)$.

(a) Graph the set of all per-period payoff combinations that are possible in a SGPNE for δ close enough to 1, according the Folk Theorem.

(b) Find a subgame perfect NE of the repeated game in which the superpowers each "Abide" in every period, on the equilibrium path. Be sure to give complete strategy descriptions.

(c) Find the smallest value of δ (as a function of α and β) such that this is indeed a subgame perfect Nash equilibrium. Analyze and interpret the comparative statics on α and β (that is, interpret the meaning and reason for the change in the minimum feasible δ as α increases and then as β increases. How do these changes affect the prospects for cooperation?).

5. *A Hobbesian answer to Locke.* A citizen and a King simultaneously choose numbers between 0 and 1, inclusive. The citizen's choice is an amount to invest in a productive but taxable activity. If the citizen invests $x \in [0, 1]$, this investment produces a total return of $2x$. The King's choice is a tax rate on the taxable resources that result (if any) from the citizen's investment. Thus if the citizen invests x and the King chooses tax rate t, the citizen gets $1 - x + 2x(1 - t)$ and the King gets $2xt$.

(a) Set this up as a normal form game and find any Nash equilibrium. What kind of problem is this?

(b) Now assume that the King can somehow commit to the set the tax rate t first, which the citizen then observes before choosing his investment. Find this new game's unique subgame perfect NE and explain the intuition behind it.

(c) Now assume that King and citizen play the normal form game in (a), but in successive periods $t = 0, 1, 2, \ldots$, and that both have a common discount factor $\delta \in (0, 1)$. Use a trigger strategy to construct an SGPNE which is socially efficient and in which (of course) the King does not confiscate everything in every period. What is the condition on δ necessary to sustain the equilibrium you propose?

(d) (extra credit) For a given δ, what is the largest per-period payoff the citizen can get in an SGPNE of the game using the trigger strategies of the last question? Does this result speak in any way to Hobbes' point against Locke's critique?