Political Science 352
Winter 2002
Problem set # 5

1. Consider the following extensive-form game: There are two players, 1 and 2. Player 1 moves first, choosing between strategies U, M, and D. If 1 chooses U, the game ends and the payoffs are (2,2). If 1 chooses M or D, then player 2 gets a move and chooses either L or R. Player 2 observes whether 1 has chosen U (and thus ended the game), but does not observe whether 1 chose M or D; 2 only knows that M or D was chosen.

   The rest of the payoffs are as follows: (M, L) implies (3, 0); (M, R) implies (0, 1); (D, L) implies (0, 1); and (D, R) implies (3, 0).

   (a) Draw the extensive form and find all Nash equilibria, SGP Nash equilibria, and perfect Bayesian equilibria of the game.

   (b) (This is a practice question that you don't need to hand in if you feel you don't have time.) Suppose that the normal form version of this game is the stage game of an infinitely repeated game with discount factor $\delta \in (0, 1)$. Characterize the set of average, per-period payoffs for 1 and 2 that can be supported in a SGPNE of the repeated game for δ sufficiently close to 1.

2. *A "No betting" result.* Ava and Bob play the following betting game. Ava is dealt a card face down from a normal pack of cards, and she looks at the card (Bob doesn't see it). Ava then chooses whether to Bet or Not Bet. If she chooses not to bet, payoffs are (0, 0). If she chooses to bet, then Bob observes her choice and chooses whether to accept the bet or not. If he rejects an offer to bet, payoffs are (0, 0). If he accepts, Ava wins the bet if the card is a red card, and loses if the card is a black card. The winner of the bet receives \$10 from the loser. Assume both players are *risk averse* in dollars, and choose appropriate utility payoffs.

   (a) Draw the extensive form of the game.

   (b) Show that there is no pure-strategy PBE in which a bet occurs, and give the intuition for this result. (If you are ambitious, show that there is no mixed strategy PBE in which a bet can occur either.)

   (c) Characterize one separating and one pooling equilibrium of the game.

3. *A very simple model of crisis bargaining.* There are two states, 1 and 2, who interact as follows. State 1 chooses whether or not to "challenge." If state 1 chooses not to challenge, both states receive their status quo payoffs (0, 0). If state 1 challenges, state 2 decides whether to resist or not. If state 2 chooses not to resist, then payoffs are (2, -7). If state 2 resists, then state 1 chooses whether or not to use force. If state 1 decides not to use force, then payoffs are (-2, 0). If state 1 fights, then payoffs are (w_1, w_2), where these are the two states' expected utilities for military conflict.

   (a) Suppose that state 2 is initially uncertain about state 1's value for war (w_1). Suppose that state 2 initially believes that $w_1 = 1$ with probability α and that $w_1 = -8$ with probability $1 - \alpha$, where $\alpha \in (0, 1)$. Represent this incomplete information problem as a game with imperfect information that has an initial move by Nature.

   (b) If state 2's initial belief is large enough – that is, greater than a particular value α^* – then this game has a pooling PBE. Find α^* and completely describe the pooling PBE. Interpret behavior in this equilibrium.

   (c) Extra credit. For $\alpha^* > \alpha$, there is a semi-separating equilibrium. Find it and interpret it.