

Solutions to Problem Set 2
Political Science 152/352

1. Assume the relation \succ on X satisfies asymmetry and negative transitivity. Let $x, y, z \in X$ and assume that $x \succ y$ and $y \succ z$. To establish the transitivity of \succ we need to show that $x \succ z$. Since $x \succ y$, negative transitivity requires $x \succ z$ or $z \succ y$. Therefore, if we can show it is not the case that $z \succ y$, we are done. But this follows from our assumption that $y \succ z$ and asymmetry of the relation \succ .

For the second part note that if negative transitivity fails, then by definition of negative transitivity, there are x, y, z such that the person has $x \succ y$ and *not* $x \succ z$ and *not* $z \succ y$. Also, *not* $x \succ z$ is consistent with $x \sim z$, and *not* $z \succ y$ is consistent with $z \sim y$. So consider $X = \{x, y, z\}$ and the preferences defined by $x \succ y$, $x \sim z$, and $z \sim y$. *not* inconsistent with transitivity of \succ but fail negativity transitivity of \succ .

2. (a) One possible set of preferences \succ on G, F and A could be $g_2 \succ g_1$, $f_2 \succ f_1$ and $a_1 \succ a_2$.

(b) $X = G \times F \times A = \{(g_1, f_1, a_1), (g_1, f_1, a_2), (g_1, f_2, a_1), (g_1, f_2, a_2), (g_2, f_1, a_1), (g_2, f_1, a_2), (g_2, f_2, a_1), (g_2, f_2, a_2)\}$ A preference relation on X should compare these 8 ordered triplets as choices. One such relation could be $(g_2, f_2, a_1) \succ (g_2, f_2, a_2) \succ (g_1, f_2, a_1) \succ (g_1, f_2, a_2) \succ (g_2, f_1, a_1) \succ (g_1, f_1, a_1) \succ (g_1, f_1, a_2)$.

(c) One ordinal utility function that represents the preferences is as follows:

$$\begin{aligned} U((g_2, f_2, a_1)) &= 10; U((g_2, f_2, a_2)) = 8; \\ U((g_1, f_2, a_1)) &= 5; U((g_2, f_1, a_1)) = 1; \\ U((g_1, f_2, a_2)) &= -3; U((g_2, f_1, a_2)) = -7; \\ U((g_1, f_1, a_1)) &= -10; U((g_1, f_1, a_2)) = -15 \end{aligned}$$

Another one would be:

$$\begin{aligned} U((g_2, f_2, a_1)) &= 200; U((g_2, f_2, a_2)) = 150; \\ U((g_1, f_2, a_1)) &= 120; U((g_2, f_1, a_1)) = 10; \\ U((g_1, f_2, a_2)) &= -150; U((g_2, f_1, a_2)) = -220; \\ U((g_1, f_1, a_1)) &= -1000; U((g_1, f_1, a_2)) = -1500. \end{aligned}$$

(d) Such a utility function should assign the same value to the alternatives in X in which the same alternative from A appear; i.e. for example it should assign the same utility value to the alternatives (g_1, f_1, a_2) and (g_2, f_2, a_2) . This is because the only relevant information for determining which alternative is better is what it picks from the set A . One such utility function could be the following: $u((g_2, f_2, a_1)) = u((g_1, f_2, a_1)) = u((g_1, f_1, a_1)) = 1$; $u((g_2, f_2, a_2)) = u((g_2, f_1, a_1)) = u((g_2, f_1, a_2)) = 0$.

(e) Set of Pareto efficient outcomes in X is the singleton $\{(g_2, f_2, a_1)\}$. This

is the most preferred outcome for the person whose preferences are specified in part (b) and also one of the most preferred for the person in (d). Therefore it is not possible to make both at least as well off and one strictly better off by way of moving to a different alternative. To see nothing else is Pareto efficient, note that for any other alternative which entails picking a_1 from the set A , moving back to (g_2, f_2, a_1) will make person in (b) strictly better off while leaving person in (d) exactly as well off. For any alternative that prescribes a_2 from the set A , moving to e.g. (g_2, f_2, a_1) will make both persons strictly better off.

3. (a) If you go to A first, with probability p you will eat at A and get a utility of a , with probability $(1-p)q$ you will have to go to B and be able to eat there, which will give you a utility of $\alpha a - c$, and with probability $(1-p)(1-q)$, both restaurants will be closed and you will get a utility of $-c$. Therefore, your expected utility from going to A first is $pa + (1-p)q(\alpha a - c) + (1-p)(1-q)(-c)$. Similarly, if you choose to go to B first, you will get an expected utility of $q\alpha a + (1-q)p(a - c) + (1-p)(1-q)(-c)$. You will prefer to go to A first if and only if the expected utility from doing so exceeds that from going to B first, i.e. $pa + (1-p)q(\alpha a - c) + (1-p)(1-q)(-c) > q\alpha a + (1-q)p(a - c) + (1-p)(1-q)(-c)$. Or equivalently, $(1-\alpha)pqa + (p-q)c > 0$.

This condition implies that if $p = q$ i.e., the restaurants are equally likely to be open, then you will surely want to try A first. You would want to try B first only if you thought it was sufficiently more likely to be open than A i.e. q sufficiently bigger than p . How much bigger q you need, depends on the costs of being wrong (c), and how much better than B you think A is (α).

(b) In substantive terms, α represents the intensity of preference for restaurant A over B . If α is close to 1, this means that the person views them as practically the same, in the sense that the person would need a very high chance of getting A in a lottery of A versus nothing so that she prefers this lottery rather than getting B for sure. If α is close to zero, this represents preferences such that the person would be willing to take a very small chance of getting into A in preference for getting B for sure. Other things equal, lower α makes it more likely that the inequality in (a) will be satisfied so the person will prefer to try A first.

(c) The values in the inequality that we may call the “utility values” are those that are multiplied by probabilities. If we multiply all utility values by K and add M to them we get the following inequality:

$$p(Ka + M) + (1-p)q(K(\alpha a - c) + M) + (1-p)(1-q)(K(-c) + M) > q(K\alpha a + M) + (1-q)p(K(a - c) + M) + (1-p)(1-q)(K(-c) + M);$$

which is equivalent to

$$K(pa + (1-p)q(\alpha a - c) + (1-p)(1-q)(-c)) + M(p + (1-p)q + (1-p)(1-q)) > K(\alpha a + (1-q)p(a - c) + (1-p)(1-q)(-c)) + M(p + (1-p)q + (1-p)(1-q));$$

or

$$K(pa + (1-p)q(\alpha a - c) + (1-p)(1-q)(-c)) + M > K(\alpha a + (1-q)p(a - c) +$$

$$(1 - p)(1 - q)(-c) + M$$

Subtracting M from both sides and dividing both sides by $K > 0$ gives us the inequality in (a).

4. (a) One example of an ordinal utility function that represents my preferences over these outcomes is $u(A) = 10, u(A-) = 9, u(B+) = 8, u(B) = 5, u(B-) = -3$.

(b) For me (Ayca), the values would be something like what follows: (i) $t = .60$; (ii) $t = .75$; (iii) $t = .85$

$$(c) u(A) = 1, u(A-) = .85, u(B+) = .75, u(B) = .6, u(B-) = 0$$

(d) I would say $t = .55$ would make me indifferent.

(e) Using the utility function in (c), the expected utility of the gamble $p = (0, t, 0, (1-t), 0)$ can be calculated as $.85t + .6(1-t) = .6 + .15t$, and when $t = .55$ as in part (d), we get that $EU(p) = .6825$. However, since in part (d) I said this gamble would make me indifferent to getting a $B+$ for sure, this expected utility should be equal to the utility of getting a $B+$, which is $.75$. Hence the answers I gave by introspection to the two parts do not seem to be consistent. However, the numbers $.6825$ and $.75$ are not very far off from each other.

5. (a) Expected issue position of candidate A is $1/3 \times (-a) + 1/3 \times 0 + 1/3 \times a = 0$. Similarly, expected issue position for B is $1/3 \times (-b) + 1/3 \times 0 + 1/3 \times b = 0$. Note, however, that candidate b 's position has higher variance, since he could end up farther away from 0.

(b) For this voter, expected utility from voting A and voting B are as follows:

$$EU(A) = 1/3 \times (-|-a|) + 1/3 \times (-|0|) + 1/3 \times (-|a|) = 2/3 \times (-a) \text{ and} \\ EU(B) = 1/3 \times (-|-b|) + 1/3 \times (-|0|) + 1/3 \times (-|b|) = 2/3 \times (-b). \text{ Since } b > a > 0, \text{ we have } EU(A) > EU(B). \text{ Therefore, this voter would vote for } A.$$

$$(c) EU(A) = 1/3 \times (-(-a)^2) + 1/3 \times (-0^2) + 1/3 \times (-a^2) = 2/3 \times (-a^2) \text{ and} \\ EU(B) = 1/3 \times (-(-b)^2) + 1/3 \times (-0^2) + 1/3 \times (-b^2) = 2/3 \times (-b^2). \text{ Since } b > a > 0, \text{ we have } EU(A) > EU(B). \text{ Therefore, the voter would still vote for } A.$$

Remark: Although the two candidates expected positions are the same, A is "less risky" in the sense that she is more likely to be close to the voter's preferred outcome. This is true whether the utility functions are linear or curved on each side of the voter's ideal point.

(d) From the above we see that among two politicians with the same expected policy issue, the one that has lower "variance" i.e. less unclarity seems

to be favored by the two voters we looked at. This seems to contradict the said observation and conjecture.

6. Below is the Social Contract Game in normal form:

		<i>Player2</i>	
		<i>V</i>	<i>NV</i>
<i>Player1</i>	<i>V</i>	industry	war at disadv for 1
	<i>NV</i>	war at disadv for 2	warre

Let U_i stand for player i 's utility function. Set

$$U_1(\text{industry}) = U_2(\text{industry}) = a$$

$$U_1(\text{warre}) = U_2(\text{warre}) = b$$

$$U_1(\text{war at disadvantage for 2}) = U_2(\text{war at disadvantage for 1}) = c$$

$$U_1(\text{war at disadvantage for 1}) = U_2(\text{war at disadvantage for 2}) = d$$

These utility values will be consistent with the Social Contract game story as long as $a > b$ (Industry is preferred to war), $b > d$ (war is preferred to war at a disadvantage), $a > c$ (Industry is preferred to war at an advantage) and $c > b$ (since war at an advantage is better than war at even odds). So altogether these imply $a > c > b > d$. If v^* is the probability that 2 plays V , then 1 is indifferent between playing V and NV if and only if $v^* \times a + (1-v^*) \times d = v^* \times c + (1-v^*) \times b$, or equivalently $v^* = \frac{b-d}{a-c+b-d}$.

If we lower 1's payoff from "war at disadvantage for 1", i.e. if we make d smaller, the v^* required to make her indifferent between playing V and NV gets bigger. Player 1, in picking her strategy, in fact is picking between two lotteries. When we lower d , the lottery she faces if she chooses NV remains the same. However, in the lottery she faces if she picks V , the "bad outcome" (war at disadvantage for 1) becomes even worse. Hence, to make her indifferent between the two lotteries, we now need a higher probability of the good outcome in the latter lottery. For values of $v < \frac{b-d}{a-c+b-d}$, best response of 1 is to play NV for sure; for $v > \frac{b-d}{a-c+b-d}$, the best response is to play V for sure; for $v = \frac{b-d}{a-c+b-d}$ both playing V and playing NV are best responses, as well as any randomization between the two.

Note that we could have set any two of these payoffs to 1 and 0, provided this got the ordinal ranking right between these two. For instance, we could have set $a = 1$ and $c = 0$, and solved for v^* in terms of b and d only. Or we could have set $a = 1$ and $d = 0$, and solved for v^* in terms of b and c . Though it can clutter up a model with notation, one advantage of using a, b, c , and d as above is that this makes it easy to do "comparative statics" on any of the utilities, such as we just did by varying d .