Solutions to Problem Set 1
Political Science 152/352

1. (a) See Figure 1. Note that this extensive form is not a unique representation of the interaction. We can change the order that players "move" and still have an equivalent representation. (The probabilities for Nature's moves are listed under the outcomes.)

   (b) Ignoring Nature, there are 3 players and 3 moves, so the normal form is $3 \times 3 \times 3$. Suppose that the three restaurants are in downtown Palo Alto: Il Fornaio, Nola, and Pluto's. Then $S_i = \{$Il Fornaio, Nola, Pluto's$\}$. An example of an element $s \in S = S_1 \times S_2 \times S_3$ is (Nola, Nola, Pluto's). In words, Friends 1 and 2 both choose Nola, while Friend 3 chooses Pluto's.

   (c) See Figure 2. Friends 1 and 2 each still have 1 information set with 3 actions each. In this modified version, since Friend 3 observes these choices, she can be at any one of $3 \times 3 = 9$ possible nodes. Since a complete strategy assigns an action for each of these nodes in the normal form and Friend 3 has three restaurants to choose from, she has $3^9 = 19,683$ possible strategies! The dimensions of the normal form are therefore $3 \times 3 \times 19,683$.

   (d) (Bonus) Writing his first preference is a dominant strategy, so if a player wishes to maximize his chances that his first preference is chosen, there are no circumstances under which he will not write it down. To see this, suppose that Player 1's first preference is Nola, and when he writes down something *other than Nola* the probability Nola is selected is $n/3$, where n is the number of other players that choose Nola. When Player 1 does write down Nola, the probability is $(n + 1)/3$. Since $(n + 1)/3 > n/3$ for any value of n, Player 1 should write down Nola to maximize the chance that this restaurant is chosen.

2. (a) See Figure 3. Let the possible outcomes from each "round" be denoted by I (industry), $1R$ (1 rich, 2 dead), $2R$ (2 rich, 1 dead), W (warre). An outcome of the entire game can be described in terms of the outcome from each round, which is done at the bottom of the figure.

   (b) There are five choice nodes for each player (one in the first round, four in the second round), and there are two available actions at each of these nodes. The dimensions of the game are therefore $2^5 \times 2^5 = 32 \times 32$. A complete strategy for Player 1 could be $F_0 F_1 P_2 F_3 P_4$. Here, I've used the subscripts to denote separate nodes where node 0 is at the top and nodes 1-4 are assigned from left to right. In words, this strategy would be "Farm in the 1st round, Farm in the 2nd round if Player 2 chose Farm in the 1st round, and Prey in the 2nd round if Player 2 chose Prey in the 1st round." Suppose Player 2 chooses the same strategy, then the outcome of the game is Industry in both rounds.

3. Define the set of possible tax rates to be $T = [0, 1]$ and let $C = \{$Compliance, No compliance$\}$. The set of strategies available to the dictator is $S_D = T \times C = [0, 1] \times \{$Compliance, No compliance$\}$. Elements of this set include (0, Compliance), a 0 % tax rate and enforced compliance, which is strange but nevertheless a valid element, and (3/4, No compliance) which means he chooses a 75 % tax that is not enforced.

4. Let $S_i = [0, 1]$ and $S = S_1 \times S_2 = [0, 1] \times [0, 1]$, which is the unit square shown in Figure 4. Elements of S include $x = (0.25, 0.75), y = (0.5, 0.5), z = (1, 0)$, which are also shown in Figure 3.

5. (a) An individual's strategy set is the set of numbers between 0 and 100, e.g. $S_i = [0, 100]$. The set of outcomes is the Cartesian product $S = [0, 100]^n$. For the case of $n = 5$, examples are $(0, 0, 10, 0, 0)$ and $(50, 60, 20, 70, 30)$.

1

(b) Assuming that a player prefers winning this game over losing, then any number $x \in S = (70, 100]$ is weakly dominated by 70. To show that x is weakly dominated by 70 we must show that for all strategy profiles that 70 produces an outcome that is just as good as choosing x, and that sometimes it is better. Let a denote the average of the numbers chosen, and let $t = 0.7a$ (e.g. 70 % of the average). Note that we get the highest value of a when every player chooses 100 so that $a \leq 100$ which implies $t \leq 70$. Then since $x > 70 \geq a$, it must be that x is always farther from t than 70 is. This means that given what others are choosing, if a player chooses x and it is the closest number to t, then if the player chooses 70 instead, it is still the closest number to t–the outcomes are just as good since in both cases the player "wins." If x is not the closest to t, then it can be that 70 is just as good when some other player chooses a y closer to t than both 70 and x (either choice "loses") or that 70 is better when 70 is closer to t than any of the other players' choices (70 "wins" when x would "lose"). (This is not precise since t will be affected by the choice of x or 70, but this is the basic logic.)

6. Let D denote the action "demand rule" and let C be "concede" so that $S_i = \{C, D\}$ for $i = 1, 2$. We can write the normal form game as:

<center>Leader 2</center>

		D	**C**
Leader 1	**D**	SoN	1 Rules
	C	2 Rules	SoN

Leader 1's preferences over outcomes are 1 Rules $\succ$ 2 Rules $\succ$ SoN. This implies that his best replies are:

- Choose D if 2 chooses C.
- Choose C if 2 chooses D.

Similarly, Leader 2's preferences over outcomes are 2 Rules $\succ$ 1 Rules $\succ$ SoN, and her best replies are:

- Choose C if 1 chooses D.
- Choose D if 1 chooses C.

There are two outcomes that are supported by mutual best replies (e.g. two Nash equilibria): 1 Rules and 2 Rules. Like the social contract game from class, there is more than one equilibrium. Unlike the social contract game, however, there is disagreement between the two leaders about which equilibrium is better. In other words, moving from 1 Rules to 2 Rules (and vice versa) does not achieve a Pareto improvement.